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Dynamic effects in optical bistability

P. Mandel† and T. Erneux‡

† *Université Libre de Bruxelles, Service de Chimie-Physique II, Campus de la Plaine, C.P.* 231, *Bruxelles* 1050, *Belgium*

‡ *Department of Engineering Sciences and Applied Mathematics, Northwestern University, Evanston, Illinois* 60201, *U.S.A.*

In the first part we give a brief review of some theoretical problems that still hinder the practical use of bistable optical devices in logic circuits. In the second part we study the influence of a time-dependent control parameter on the solution of differential equations. We prove that even for small sweeping rates large delays can be expected, resulting in the dynamical stabilization of unstable solutions. Examples are given for dynamically induced optical bistability in a laser and dynamical stabilization in a laser with saturable absorber.

Introduction

Optical bistability (o.b.) has been for many years a stable attractor for theoretical and experimental physics (Bowden *et al.* (eds) 1981, 1984; Abraham & Smith 1982; Lugiato 1984). It has now reached a development such that one begins to think seriously of using bistable optical devices ('biodes') to construct all-optical logic circuits and hence all-optical computers. The topics covered by this Discussion Meeting illustrate in many respects the state of the art in this domain. In the first part of this paper, we review some of the theoretical problems that still hinder the practical realization of an all-optical computer (for each topic we have selected a few references; no attempt has been made to provide an exhaustive list of references). In the second part we study more carefully one problem, related to the time-dependence of the input field.

Review

Bistability is a property that is often found in Nature as soon as the phenomena involve a nonlinear mechanism. Electronic circuits, chemical reactions, hydrodynamics, biological systems and quantum optics display quite a number of situations in which bistability has been predicted or observed, or both. In all-optical arrangements, o.b. can be achieved by using either active or passive systems. The passive system most widely studied (Miller *et al.* 1979; Gibbs *et al.* 1979) is a Fabry–Perot resonator pumped by a coherent beam; inside the cavity a semiconductor provides the necessary nonlinearity. O.b. can also be realized in active systems in which the inversion of a population is created by some incoherent mechanism. In fact the earliest proposal to create a biode is due to Lasher (Lasher 1964), who suggested coupling two semiconductor lasers in a single cavity to make a laser with saturable absorber. In such a system one part of the device is acting as a normal laser whereas the other part provides the nonlinear mechanism. This laser with a saturable absorber displays o.b. and passive Q-switching (Harder *et al.* 1982).

O.b. refers to a situation where two stable states coexist, implying a hysteresis effect. Each of these two states can be steady or periodic. Although chaotic attractors have also been found and are very important for fundamental research, they are of little interest for the topics of this Discussion Meeting. One way to achieve o.b. is to have a resonator whose length is a function of the input intensity. This can be achieved, in passive systems, by allowing an intensity dependence of either the physical length or the optical length of the resonator. In the first case, one uses the pressure of the input radiation to modify the position of a suspended mirror (Dorsel *et al.* 1983), the cavity being empty. In the second case the mirrors are fixed but the cavity contains a medium whose complex susceptibility is significantly intensity-dependent (i.e. a nonlinear medium). When the input field, the material medium and the cavity have nearly equal frequencies, the dominant mechanism will be nonlinear absorption (i.e. absorptive o.b. (see Bonifacio & Lugiato 1976; Szöke *et al.* 1969; Weyer *et al.* 1981)). When there are large detunings we can still observe o.b. (in this case dispersive o.b.) involving nonlinear refraction coupled to linear absorption (Gibbs *et al.* 1976; Marburger & Felber 1978). The physics of absorptive and dispersive o.b. are quite different and therefore lead to different experimental constraints and optimization schemes. Nevertheless these schemes show a number of problems that need to be resolved if we wish to build an all-optical computer. Some of these problems have been carefully analysed by Fork (Fork 1982). Let us consider a thin sample of nonlinear material with a large section. A number of beams at constant intensity are aimed at the sample and at each entry point a probe beam or pulse is added to obtain a logic operation by means of o.b. in the bulk of the sample (Smith 1984). The obvious main constraints to be imposed are that each pixel (in this case the volume used by the beam within the sample) should be stable before and after the switching and should be independent of its neighbours. The following factors, to be discussed at this meeting, affect the stablity of and the cross-talk between pixels.

(i) *Transverse effects.* A real input beam has a finite diameter and a transverse intensity profile. This affects both steady-state and dynamic responses of a biode. At steady state the width of the hysteresis domain is generally reduced (Drummond 1981) and can even vanish, owing to transverse effects. Moreover the domain for stable steady states is usually reduced by transverse effects (Moloney *et al.* 1982). More important is the dynamics of the switching process, which is deeply affected because not all parts of the beam will switch simultaneously (Rosanov & Semenov 1981). The resulting increased gradient of intensity may enhance the self-focusing or defocusing in the beam. Transverse effects will be discussed by Lugiato & Narducci at this meeting (see also Firth & Wright 1982).

(ii) *Diffusion.* Another limitation to dense packing of pixels is diffusion cross-talk. It is characterized by the recombination time of the free carriers, to which the nonlinear refraction index is directly proportional. If N is the density of carriers, the recombination time is proportional to N^a, depending on the dominant mechanism through which the carriers recombine, e.g. the direct trapping of carriers yields $a = 1$, the Auger process yields $a = 3$ (direct three-body interaction) and radiative recombination gives $a = 2$. The avoidance of diffusion cross-talk requires a good understanding of the microscopic mechanism on which o.b. is based. This question is discussed in details by many authors in this meeting.

(iii) *Diffraction.* Another source of cross-talk is diffraction. It is intimately connected with transverse effects and competes with diffusion against dense packing of pixels. It is already better understood than diffusion because it does not involve a refined understanding of the light–matter interaction and can therefore be treated at a macroscopic level. In a nonlinear medium it will induce self-focusing or defocusing, both effects being potentially disastrous. The relative weight of diffraction and diffusion is analysed by Firth and his colleagues at this meeting.

(iv) *Noise*. A pixel is created by the presence of a holding beam with an intensity located in the bistable domain. This domain may be fairly small, or we may wish to have a holding intensity near the switch-up point, or both. In both cases the stability of the biode will be affected by unavoidable noise in the holding beam. Here too a knowledge of the noise tolerance is required to realize a reliable biode (Schmidt *et al.* 1983; Willis 1983; DelleDonne *et al.* 1981). Noise problems are considered by Arecchi at this meeting.

(v) *Dynamics*. There are relatively few results on the dynamical properties of biodes because their discussion requires a solution of time-dependent equations. Most results are therefore numerical or experimental. Among the dynamical properties let us mention:

the dependence of switch-up and switch-down times upon the cavity and atomic decay rates (Mandel & Erneux 1982; Erneux & Mandel 1983; Moloney & Gibbs 1982; Hopf & Meystre 1979; Bonifacio & Meystre 1978; Bischofberger & Shen 1979);
critical slowing down (Bonifacio & Meystre 1979; Garmire *et al.* 1979);
overshoot switching (Goldstone *et al.* 1981);
self-pulsing (Bonifacio & Lugiato 1978; Lugiato & Milani 1983).

Although some of these topics are reviewed at this meeting for specific materials, a reasonable theory has still to be worked out.

Bifurcations with time-dependent parameters

In most problems that we face in quantum optics, there occur bifurcation points at which two states coalesce. The three most common critical points are:

(i) steady bifurcations, where a stationary state emerges from another steady state;
(ii) Hopf bifurcatons, where a time-periodic state emerges from a steady state;
(iii) limit points, where a solution ceases to exist.

For technical reasons, it is often necessary to investigate these bifurcation points experimentally by sweeping a suitable parameter (the bifurcation parameter) across the transition domain. It is usually argued that if the sweeping rate is small enough, the dynamical effects associated with the time-dependence of the bifurcation parameter will be negligible and the system will somehow 'adiabatically follow' the states described with a constant bifurcation parameter. We shall see that this assumption may be quite wrong.

Let us first take a pedagogical example. Consider the equation

$$z_t = zA - z^2, \tag{1}$$

which corresponds to the cubic approximation of the standard laser equations when the atomic variables have been adiabatically eliminated. Here z is the field intensity and A is the pump parameter plus one. When A is constant, (1) has two steady states: $z = 0$ and $z = A$. The trivial solution $z = 0$ is stable for negative values of A whereas $z = A$ is stable for positive A. Hence $A = 0$ is a steady bifurcation point.

When A is time-dependent, the situation is quite different. First we note that even if $A_t \neq 0$, the trivial state $z = 0$ remains an exact solution of (1). Second, if the initial condition verifies the inequality $z(0) = z_i \ll 1$, we can analyse the stability of $z = 0$ by linearizing (1) around the trivial solution

$$z_t = Az. \tag{2}$$

The solution of (2) is

$$z(t) = z_i \exp \int_0^t A(s)\,\mathrm{d}s. \tag{3}$$

It is clear that (3) will diverge (i.e. z will become unstable) when

$$\int_0^{t^*} A(s)\,\mathrm{d}s = 0, \tag{4}$$

which is the dynamical bifurcation equation. If we define $\bar{t}$ as the time at at which the steady bifurcation is reached, i.e. $A(\bar{t}) = 0$, then we can decompose (4) into

$$\int_0^{\bar{t}} A(s)\,\mathrm{d}s = -\int_{\bar{t}}^{t^*} A(s)\,\mathrm{d}s, \tag{5}$$

which expresses the balance between the stability accumulated from 0 to $\bar{t}$ (where $A(s)$ is negative) and the instability accumulated from $\bar{t}$ and t^* (where $A(s)$ is positive). An obvious inequality is $t^* > \bar{t}$, implying that $A(t^*)$ is necessarily delayed compared with the steady value $A(\bar{t})$. For a linear dependence,

$$A(t) = A(0) + bt, \quad A(0) < 0, \quad b > 0, \tag{6}$$

it is elementary to solve (4) and to find

$$A(t^*) = -A(0). \tag{7}$$

In other terms, the distance between the dynamical and the steady bifurcations $(A(t^*) - A(\bar{t}))$ is equal to the distance between the steady bifurcation and the initial value $(A(\bar{t}) - A(0))$ and independent of the sweeping rate, b. This counterintuitive result is a long way from the assumed adiabatic following of the steady states.

Of course, (7) was derived under the assumption that we may neglect the nonlinear term in (1). We can remove this limitation by studying the exact solution of (1), which is

$$z(t) = \frac{\exp \int_0^t A(s)\,\mathrm{d}s}{z_i^{-1} + \int_0^t \left\{\exp \int_0^{t'} A(s)\,\mathrm{d}s\right\} \mathrm{d}t'}. \tag{8}$$

For constant A this reduces to

$$z(t) = A\{(A/z_i)\,\mathrm{e}^{-At} + 1\}^{-1}.$$

The solution (8) gives qualitatively the same results as (7) as long as $0 < b \ll 1$.

We now give two examples where delayed bifurcaton leads to new effects.

(i) *Transient. o.b.* We first consider the semi-classical laser equations

$$\begin{aligned} E_t &= -E + Av, \\ v_t &= d(-v + EF), \\ F_t &= d_{\parallel}(-F + 1 - Ev), \end{aligned} \tag{9}$$

for which a similar analysis has been performed (Mandel & Erneux 1984). In figure 1 we indicate the result of a numerical integration of (9) with $d = d_{\parallel} = 10$, $A(t) = -0.5 + 10^{-2}t$. As

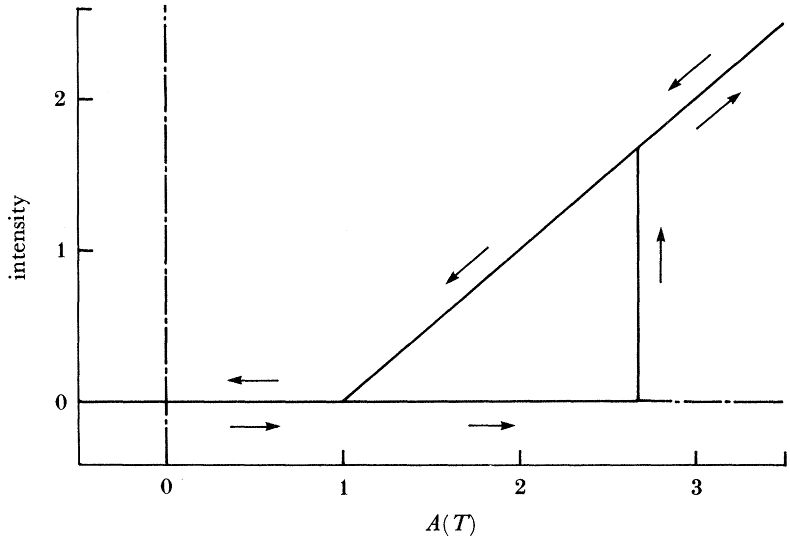

FIGURE 1. Dynamical hysteresis for a laser.

expected from our simple example, the zero intensity state is dynamically stabilized, but there is no significant change of stability for the solution $E^2 = A-1$. Therefore making a forward sweep followed by a backward sweep will induce two different responses and we create by this mechanism a transient bistable response. Transient o.b. is not new and has been predicted in optics in a different context by Broggi & Lugiato (1984).

(ii) Another example is taken from our study of o.b. in a laser with saturable absorber (l.s.a.) (Erneux & Mandel 1984) and is shown in figure 2. There we have solved the eight semiclassical equations for an l.s.a. and let the pump parameter decrease, starting from the upper branch. For constant A, the linear stability analysis indicates that the upper branch has a Hopf bifurcation at $A = 4.2$ (indicated by a circled bar on the curve), which is reached for $t = 260$. When $A < A_H$ (i.e. $t > 260$) the steady upper branch is unstable. However, owing to the time-dependence of A, the dynamical bifurcation is delayed. In this precise example, the delay is simply larger than the time necessary to reach the endpoint of the upper branch where the jump to the trivial solution occurs.

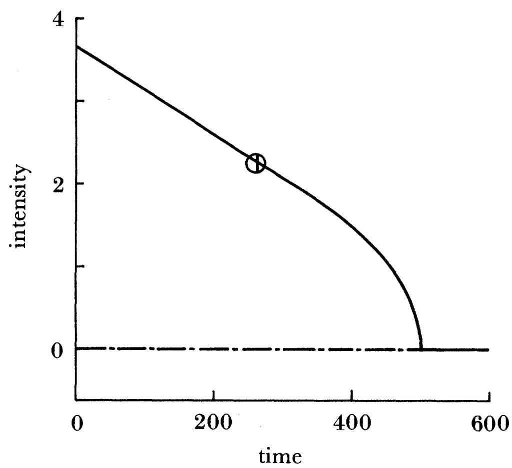

FIGURE 2. Dynamical stabilization: for $t > 260$ the upper branch is unstable with a constant pump parameter and stable for a time-dependent pump parameter.

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We are now investigating the same type of effects on intrinsic o.b., and preliminary results indicate that delayed bifurcations do occur for the upper and the lower branches. Contrary to most effects discussed in the first part of this paper, the time-dependence of the control parameter results in the widening of the hysteresis domain.

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